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Fostering Computational Thinking and ICT Integration in Mathematics Teacher Education: A Two-Cycle Study in Portugal.

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Abstract

Computational thinking is mandated as a cross-curricular requirement in the current Portuguese mathematics curricula from primary to secondary levels, and the guidelines advocate the use of technology. Official documents propose strategies to foster computational thinking through dynamic geometry environments, including GeoGebra, internet applets, Scratch, and Python. This study examined how prospective teachers interpret these tools, focusing on first-year students enrolled in the Master's programme in Teaching Mathematics for Basic and Secondary Education, a course with a long tradition in preparing mathematics teachers to use technological pedagogical tools. The work of twelve students was analysed through a design-based research methodology, attending to their activities over two academic years and the adjustments made between the first and second intervention cycles. Qualitative analysis indicates that integrating content knowledge with technological knowledge is complex. The inclusion of pedagogical content knowledge alongside these domains poses further challenges in initial teacher education. The study offers insights for refining the course unit in subsequent iterations and identifies considerations for ongoing teacher professional development in Portugal.
Keywords: *Mathematics Curriculum, Computational Thinking, Technology Education, Pre-service Teacher Training*

1 Introduction

Considerations regarding the integration of Information and Communication Technologies (ICT) into mathematics education can be traced to 1985. The mathematician Jean-Pierre Kahane, then President of the International Commission on Mathematical Instruction (ICMI), initiated the first ICMI Study on the use of informatics in mathematics teaching because "at that time it seemed evident that informatics was likely to have an important influence on mathematics education but many professional mathematicians were not already convinced that informatics would have a substantial influence on their mathematical practices" [1, p. 463–464]. A second ICMI Study was launched twenty years later, mainly because no one would deny the influence of informatics and digital technologies on the professional practices and life of mathematicians"; yet, in mathematics education "the situation is not so brilliant and no one would claim that the expectations expressed at the time of the first study have been fulfilled" [1, p. 464].

Since the beginning of the twenty-first century, the integration of ICT into mathematics education has been reported to benefit teachers and learners by increasing motivation and performance and by supporting lifelong learning [2, 3, 4].

Official Portuguese mathematics curriculum guidelines, in force since 1991, have progressively incorporated technological tools: initially scientific calculators, later graphing calculators, and currently a systematic use of digital resources (excluding computer algebra systems). The most recent documents explicitly refer to computational thinking and to the use of programming algorithms [5, 6]. Computational thinking (CT) now functions as a cross-curricular theme, and the guidelines recommend the systematic use of dynamic geometry environments, programming tools, and internet applets to enrich mathematics instruction [7, 8, 9].

The shift from traditional methods to technology-mediated pedagogies positions learners in more active roles, although difficulties related to teacher preparation, technical support, and limited ICT knowledge remain [2, 3]. Recent studies indicate the potential of CT to strengthen mathematical reasoning and problem solving, equipping students with competences required in a digital era [10, 11, 12]. Teachers, however, meet obstacles when integrating CT, including inadequate preparation and unfamiliarity with programming, pattern recognition, and algorithmic approaches [13, 14]. Addressing these challenges calls for strategies that develop the capacity of future mathematics teachers to use such tools in their practice.

This study investigates the perceptions and practices of twelve pre-service teachers enrolled in the first year of a Master's programme in Teaching Mathematics for Basic and Secondary Schools in Portugal. Their work in a *Computational Means in Mathematics Education* course was examined through a design-based research approach over two academic years. The analysis exposes the complexities of aligning content knowledge, technological skills, and pedagogy, suggesting that developing CT within mathematics curricula introduces novel demands for teacher education. The paper discusses implications for refining the course unit and identifies considerations for continuing teacher professional development in Portugal.

2 Framework

In this section, a review of theoretical foundations is provided to situate the investigation. The discussion centres on the integration of computational thinking and technological tools in the preparation of in-service and pre-service mathematics teachers, drawing on several established models and theories.

2.1 Rogers' Model of Innovation in Education

Rogers' diffusion of innovations theory [15] offers a conceptual lens for examining how educators adopt new technologies. The model describes successive stages—knowledge, persuasion, decision, implementation, and confirmation—that occur when an innovation is introduced into instructional settings. Teacher beliefs and attitudes influence each phase of the adoption process. Although professional organisations, such as the National Council of Teachers of Mathematics, have long promoted technology integration, scholars have identified a marked gap in published research addressing technology-focused professional development in mathematics education between 1975 and 2015 [1]. This gap limits teachers' opportunities to acquire the knowledge needed to integrate ICT.

Over the same period, lesson study gained recognition as a productive approach for collaboration, reflection, and the improvement of mathematics teaching [16, 17, 18]. Broader initiatives, including cluster models and large-scale continuing education programmes, have been successful when teacher training aligns with local leadership and collective efforts [19, 20].

Pre-service teacher education has likewise benefited from the purposeful use of a variety of technologies [21, 22]. Such experiences foster favourable attitudes among prospective teachers, enabling them to envisage digital tools as means of enriching mathematical understanding rather than of reinforcing established practices [23]. Researchers note, however, that bridging pedagogical content knowledge and technological capabilities remains demanding [24, 25, 16, 26]. Sustained innovation appears to require extended support and collaboration among educators, technical staff, and mentors, together with alignment with organisational leadership [27]. Efforts to link mathematics teacher education to Education for Sustainable Development further underscore the need for contextualisation and for competencies that connect mathematics with environmental concerns [28]. UNESCO observes that mathematics underpins sustainable development and provides an essential foundation for critical citizenship and lifelong learning in a rapidly changing world [29, p. 15]. It accordingly argues that teachers must possess awareness of connections with more advanced mathematical ideas, such as mathematical modelling [29, p. 16].

2.2 Instrumental orchestration in pre-service mathematics teachers

Instrumental orchestration refers to teachers' systematic use of technological tools to organise classroom activity and guide students' learning [30]. The concept has evolved through research that examines how teachers manage instructional complexity, design resources, and facilitate interaction with digital environments [31]. Drijvers etal. identified several whole-class orchestration strategies, including *Technical-demo* and *Discuss-the-screen*, which enable teachers to manage technology-based tasks effectively and to promote mathematical understanding [32]. More recently, nine forms of orchestration have been distinguished (Table 1). Qi Tan and Zhiqiang Yuan [33] subdivide the *Work-and-walk-by* orchestration [34] into categories such as *Technical-demo*, *Guide-and-explain*, *Link-screen-paper*, *Discuss-the-screen*, and *Technical-support*.

Recent studies have adapted these approaches for online contexts and have extended the framework, for example through instrumental meta-orchestration, within teacher education [35, 36, 37]. Integrating instrumental orchestration into training programmes obliges pre-service teachers to plan coherent lessons, align technological resources with curricular objectives, and evaluate pupil work in real time. Research on dynamic software, including GeoGebra, indicates that thorough integration enables teachers to broaden opportunities for active exploration of mathematical concepts [38, 33, 34, 39]. Instrumental orchestrations are often employed in ICT-supported settings, particularly those involving GeoGebra [39, 40].

Table 1: Descriptions of nine whole-class orchestrations (adapted from Drijvers et al. [34] in [33, p. 13])

Orchestration	Description
Technical-demo	The teacher demonstrates tool techniques.
Explain-the-screen	The teacher explains mathematical content to the whole class, guided by what appears on the screen.
Guide-and-explain	The teacher poses closed questions based on the screen, with interaction so limited and guided that it cannot be regarded as an open discussion.
Discuss-the-screen	The whole class discusses what is happening on the screen.
Link-screen-board	The teacher emphasises the relationship between the technological environment and representations in conventional media (paper, book, blackboard).
Spot-and-show	The teacher foregrounds student reasoning by selecting notable work produced in digital environments.
Sherpa-at-work	A pupil (the "Sherpa") uses the technology to present work or to execute actions requested by the teacher.
Board-instruction	The teacher teaches the whole class in front of the board, which is used solely for writing.
Work-and-walk-by	The teacher circulates through the classroom, offering tailored guidance to individuals or groups.

2.3 Integrating technology in teaching mathematics within a favourable curriculum

International research has investigated how technology adoption in mathematics education impacts pupil achievement, teacher professional development, and classroom practice [40]. Findings suggest that continuing professional support and the development of teacher identity facilitate effective ICT use [41]. New pedagogical approaches, however, frequently demand additional planning, time, and materials, which some educators regard as burdensome [42, 43].

In Portugal, the development of computational thinking is not confined to a single subject, yet mathematics courses provide the clearest guidance on its use. The current official syllabus combines a unified mathematics course for Basic Education (Years 1–9) with a diversified set of courses for Secondary Education (Years 10–12).

For Basic Education, the syllabus (approved 2021) stipulates that "all students must be able to access calculators, robots, internet applications, and software for statistics, geometry, functions, modelling, and visual programming environments" in order to "promote more meaningful learning and broaden the contexts in which pupils engage with mathematical objects" [5].

For Secondary Education, the syllabus (approved 2023) calls for the "systematic use of technology" to encourage "the exploration of ideas and concepts, using technology as a lever for understanding and solving problems" [6].

Thus, in Basic Education pupils are expected to "develop and mobilise computational thinking" [5], while in Secondary Education a wide range of resources is recommended to foster "algorithmic processes, structured thinking, and logical reasoning", thereby "genuinely involving problem formulation and solution and fostering computational thinking" [6, 29].

Recent curriculum guidelines, therefore, encourage the development of computational thinking across essential mathematics standards from primary to secondary levels [44]. They enable mathematics teachers to employ new technologies, such as programming environments and applications, across multiple topics.

Exposing pre-service teachers to these experiences is critical, as they often enter teacher education programmes with beliefs shaped by their own schooling. Systematic integration of technology throughout the mathematics curriculum and a deliberate focus on computational thinking necessitate careful design and intentional scaffolding during teacher preparation. Yet only a small proportion of pre-service teachers devise lesson plans that reflect technological, pedagogical, and content knowledge (TPACK) by balancing mathematical reasoning, technological knowledge, and appropriate pedagogy [45]. This study therefore seeks to understand the challenges and benefits that arise when teacher-education curricula are devised to foster these intersections in ways aligned with national requirements.

2.4 Historical overview of the course "Computational Means in Mathematics Education" (MCEM)

This second-semester unit, *MCEM*, directed to prospective mathematics teachers, was introduced in 1987. Initial teacher preparation has always extended over five years, even before the current structure of bachelor (three years) plus master (two years). Although the content has evolved, the goal remains: "If future teachers become comfortable doing mathematics with the computer, then they will be prepared to use it in their ordinary teaching" [46]. Programming languages are studied from the perspective of using computers in the teaching of mathematics and in relation to applications and mathematical modelling. LOGO was the principal language for a number of years. The course has moved from exclusive reliance on computers, through the introduction of graphing calculators, to the present inclusion of 3-D printers and educational robots. The software employed has been varied and has included David Smith's *MATHPROGRAM* and Harley Flanders's *MICROCALC*. Dynamic geometry software has been adopted as soon as it became available, beginning with *Cabri-Géomètre* and now GeoGebra.

3 Methods

This study employed a design-based research (DBR) methodology, which is appropriate when the intention is to examine educational interventions in authentic settings and iteratively refine pedagogical strategies based on empirical evidence. Design-based approaches allow for the integration of theory with practice, and have been recommended for investigating complex learning environments involving digital tools and teacher education [47, 48]. Within the scope of this inquiry, the DBR framework served to examine and adjust the pedagogical design of a course unit focused on computational tools in mathematics education.

The course *Computational Means in Mathematics Education* (MCEM) is preceded by two units on specific didactics of mathematics and a unit on the history of mathematics; in the same semester students also attend two further modules on didactics. The present investigation covered two academic years: 2022–2023 (Cycle 1) and 2023–2024 (Cycle 2). It addressed two research questions: (i) How do students, as prospective teachers, perceive the use of technology in the teaching and learning of mathematics? (ii) Which strategies foster more effective practice?

In each cycle, students' work in MCEM was examined. Cycle 1 comprised eight projects (ST1–ST8); Cycle 2 comprised four (ST9–ST12). During both years students engaged with technology in first-semester modules on analysis and geometry didactics, using GeoGebra, Scratch, Python Blocks, graphing calculators, ASYMPTOTE, and MathCityMap. The tasks were designed for application in primary and secondary classrooms. All technological engagement was embedded within tasks derived from national curricular expectations and aligned with learning objectives for both primary and secondary education. The selection of these tools and the nature of the tasks correspond to current policy guidelines in Portugal, which mandate the development of computational thinking and encourage the integration of technology as a cross-curricular competency [5, 6].

In the first academic cycle (2022–2023), participants were individually responsible for creating structured digital portfolios, incorporating a range of activities combining artistic, cultural, and mathematical content. These portfolios were examined qualitatively to determine how students articulated the intersection of technological and mathematical understanding. Portfolios was presented in a web page created by each participant, containing: (i) a brief academic profile; (ii) evidence of completion of a MOOC; (iii) examples linking mathematics and art drawn from `https://www.europeana.eu/pt`; (iv) a mathematical cartoon for International Mathematics Day; (v) software they intended to employ as teachers; and (vi) a trail designed with MathCityMap; (vi) completed collaboratively, the remainder individually. The resulting pages were analysed and informed modifications to the Cycle 2 assignment. The format of these assignments was informed by the theoretical perspectives of technological pedagogical content knowledge (TPACK)[24], which address the confluence of content, pedagogy, and digital tools in teacher preparation. However, limitations in the coherence and depth of technological integration were observed. In particular, the capacity to align computational thinking with curricular objectives and lesson design was found to be underdeveloped among several participants. These observations substantiated the need for more structured collaborative and reflective components in the subsequent iteration.

Accordingly, the second cycle (2023–2024) adopted a refined approach, consistent with the iterative logic of DBR. Participants worked in dyads to develop thematic lesson portfolios and to conduct simulated classroom sessions. Each dyad addressed one designated topic and developed lesson sequences grounded in computational tools — for example, Topic 2: visualisation with Python in real and complex analysis (upper secondary); Topic 3: Scratch for geometry and algebra (essential learning). The portfolio included (a) collaborative materials and (b) individual lesson-planning and reflection tools. Each student delivered two 45-minute simulated lessons, observed by a course instructor and peers. Post-lesson, a discussion was followed by completion of an evaluation rubric known to all participants in advance. Thematic working sessions supported the preparation of plans (see Table 2, Appendix). The instructional simulations were presented in 45-minute sessions, which were observed by peers and course instructors. Evaluative discussions followed each session, supported by pre-defined rubrics made available beforehand. These rubrics focused on the integration of digital tools, clarity of mathematical reasoning, anticipatory teaching strategies, and classroom orchestration. This structure reflects established practices in lesson study and research on professional noticing in teacher education

[16, 18].

The data collection for both cycles occurred after the conclusion of coursework and summative evaluation. The analysis focused on the lesson designs, teaching simulations, and reflective components provided in the portfolios. Ethical protocols were observed in the anonymisation and handling of all data.

This methodological structure aligns with calls in the mathematics education literature for approaches that embed professional learning within authentic and sustained contexts, particularly when digital technologies and computational thinking are involved [21, 32].

4 Results

4.1 First cycle

The work of the eight students (ST1–ST8) reveals recurrent themes in their use of computational tools and in the projects developed for the final assignment. Each student constructed a web page containing the required elements; the content analysed is summarised below.

- **ST1** employed GeoGebra, Desmos, Kahoot, and Poly, selected visual materials from *Europeana*, and produced a comic to mark International Mathematics Day.
- **ST2** focused chiefly on Poly, created mathematics-related comics and images, and completed an online course on active learning.
- **ST3** used Desmos, contributed to the *Europeana* project, created a comic, and completed an eTWINNING course.
- **ST4** integrated GeoGebra, Desmos, Kahoot, and Poly, contributed to *Europeana*, developed a MathCityMap trail, and undertook social-network training.
- **ST5** combined GeoGebra, Desmos, Kahoot, and Poly, engaged with *Europeana*, and created a comic.
- **ST6** presented applications employing the same four tools, contributed to *Europeana*, and completed an eTWINNING course.
- **ST7** extended application by designing a new MathCityMap resource, creating a comic, and drafting a detailed lesson plan incorporating Python.
- **ST8** produced detailed lesson plans centred on Python and contributed to *Europeana* and MathCityMap, alongside participation in an eTWINNING project.

Across the cohort, students demonstrated the capacity to integrate multiple ICT tools—GeoGebra, Desmos, Kahoot, Poly, and video resources—into prospective teaching practice. Participation in interdisciplinary initiatives such as *Europeana* and MathCityMap suggests an intention to employ real-world contexts. The creation of comics and involvement in eTWINNING indicate creative and collaborative engagement. Completion of online courses (eTWINNING, NAU) evidences commitment to continuing professional development. The lesson plans produced by ST7 and ST8, which incorporate Python, represent a deeper integration of computational thinking within mathematics education.

4.2 Second cycle

Given the aims of the study, the analysis concentrated on the two dyads that produced portfolios: ST9&ST10 and ST11&ST12. Each portfolio documents collaborative planning and individual reflection centred on the use of specific computational tools—Python in the first dyad and Scratch (with GeoGebra) in the second.

4.2.1 Portfolio produced by dyad ST9&ST10

The collaborative section justifies lesson plans on the following topics: ST9 — "Functions Defined by Branches" and "Cubic and Quartic Functions"; ST10 — "Quadratic Functions" and "Complex Numbers". Individual reflections evaluate the simulated lessons, drawing on peer and supervisor feedback.

The sequence of lessons progresses from elementary to more advanced content, beginning with block-based programming (EduBlocks) to introduce Python and moving to activities that require text-based coding. Figure 1 illustrates an introductory task on quadratic functions.

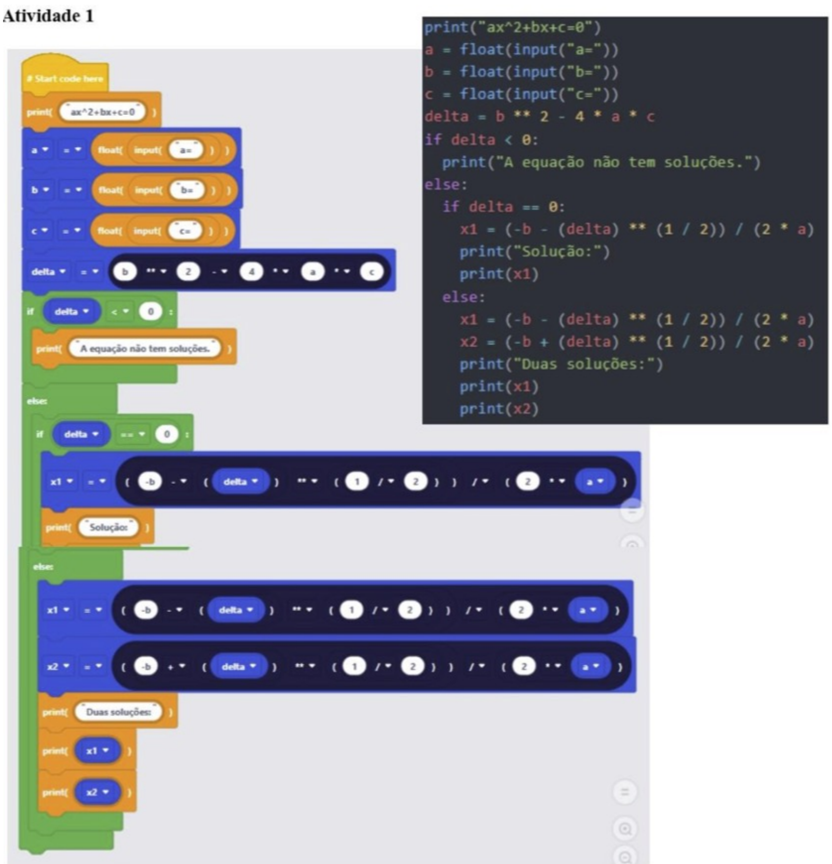

Figure 1: Task 1 of the first simulated lesson by ST10; full solution included in the lesson plan.

The plans integrate technological tools (Python), pedagogical strategies (graded complexity, visual aids), and mathematical content (functions, polynomial equations, complex numbers). Hypothetical learning trajectories anticipate difficulties—e.g. defining piecewise functions—and suggest targeted interventions. Reflections acknowledge the need to align computational objectives with mathematical aims and to provide precise programming guidance.

Overall, the portfolio demonstrates thoughtful use of Python to support computational thinking, algorithmic reasoning, and logical problem-solving.

4.2.2 Portfolio produced by dyad ST11&ST12

The collaborative component outlines lesson plans that employ Scratch (and, for ST11, GeoGebra) to teach Year 8 topics. ST11 addresses linear functions; ST12 develops lessons on the Pythagorean theorem and isometries.

For ST11, Scratch scripts and GeoGebra applets are combined to promote exploration of gradients and intercepts through guided discovery and collaborative problem-solving. For ST12, Scratch scenarios place the Pythagorean theorem in narrative contexts and use the Scratch cat to discuss planar movements; Figure 2 shows two examples.

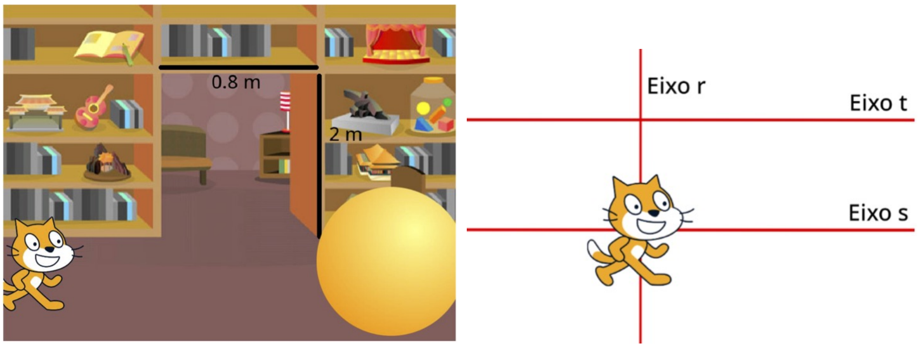

Figure 2: Scratch scenarios employed in the lesson plans of ST12.

Hypothetical learning trajectories identify likely misconceptions (e.g. proportional reasoning in linear functions) and propose scaffolds. Reflections discuss the efficacy of interactive tools in fostering engagement and reasoning and recognise areas for refinement suggested by peers and supervisors.

4.2.3 Computational thinking across the two portfolios

Dyad ST9&ST10. Computational thinking is embedded through:

- *Algorithmic processes*: explicit step-by-step procedures in Python.
- *Decomposition*: division of complex tasks into manageable sub-tasks.
- *Abstraction*: modelling mathematical objects within code.
- *Automation*: use of scripts to perform repetitive calculations and visualisations.
- *Data representation*: dynamic displays of functions and loci.

Dyad ST11&ST12. Computational thinking is advanced via:

- *Interactive problem-solving*: Scratch projects requiring iterative refinement.
- *Visualisation*: GeoGebra and Scratch animations to render abstract notions.

- *Decomposition and abstraction*: stepwise modelling of geometrical situations.
- *Pattern recognition*: identification of regularities through multiple Scratch scenarios.
- *Iterative development*: debugging and improving scripts in response to feedback.

Both portfolios confirm that well-designed computational activities can deepen learners' mathematical understanding. Whereas ST9&ST10 prioritise algorithmic structures and systematic coding, ST11&ST12 emphasise interactive exploration and visual modelling. In each case the pre-service teachers exhibit reflective practice and a willingness to refine instruction in response to evidence.

5 Discussion

The purpose of this section is to interpret the findings from both research cycles in light of the conceptual framework and the existing literature on teacher education, computational thinking, and the integration of technology into mathematics pedagogy. The discussion is organised to examine the development of professional knowledge, the integration of computational thinking, and the orchestration of digital tools in simulated practice.

5.1 Concerning to 1st Cycle

The first cycle revealed partial engagement with technological and computational tools, despite the availability of digital resources and the inclusion of computational thinking within national curricular documents [5, 6]. While the students demonstrated familiarity with applications such as GeoGebra, Desmos, and MathCityMap, only a minority formulated teaching sequences that showed alignment with the principles of technological pedagogical content knowledge (TPACK) [24]. Most submissions were task-oriented and exploratory rather than grounded in pedagogical intent. The absence of collaborative planning and structured simulation likely limited opportunities for participants to connect digital tools with instructional strategies in a sustained manner. This finding is consistent with previous studies indicating that short-term exposure to technology does not suffice to support the formation of integrated teaching knowledge [25, 23] .

The redesign implemented in the second cycle introduced collaborative planning, structured simulation of lessons, and systematic reflection. The resulting portfolios demonstrated improved alignment with curricular aims and more deliberate integration of computational thinking strategies, including abstraction, algorithmic reasoning, and decomposition. These developments are in line with international recommendations for supporting novice teachers through iterative design, critical reflection, and peer collaboration [42, 49].

5.2 Concerning to 2nd Cycle

A more refined approach was introduced in the second cycle. Students worked collaboratively on lesson design that applied programming and dynamic environments to mathematical content, which reflected recommendations from teacher education research [1, 20]. Each pair produced a reflective portfolio containing carefully structured lesson plans and analyses of simulated classroom activities. This design aligned with the call for systematic teacher professional development to address the complexity of integrating new technologies [24, 16].

5.2.1 Results of ST9&ST10 dyad

The collaborative efforts of ST9 and ST10 reflect a methodical approach to teaching advanced topics in secondary mathematics, specifically functions and complex numbers. Their lesson plans demonstrate a clear alignment with curriculum standards and a thoughtful justification for the selection of topics. This aligns with Rogers' Diffusion of Innovations theory [15], which highlights the importance of understanding how new educational practices and technologies are adopted within teaching contexts. By incorporating Python programming into their plans, these students exemplify a commitment to integrating innovative tools into their teaching practices.

The structured progression from simpler functions to more complex concepts illustrates the pedagogical strategies outlined in the TPACK framework [24]. The inclusion of hypothetical learning trajectories indicates a proactive approach to addressing potential student misconceptions, which is essential for effective teaching practice. For instance, ST9 and ST10's focus on the vertex of piecewise functions as a critical learning point aligns with educational models that advocate for anticipatory teaching strategies.

The portfolio of dyads ST9 & ST10 also reflect the principles of instrumental orchestration, particularly the Technical-demo and Guide-and-explain orchestrations. By demonstrating Python techniques and guiding students through complex mathematical concepts, they enhance students' understanding and engagement. This approach is consistent with findings that emphasize the importance of careful planning and arrangement of digital artifacts in teaching environments [37].

Also, The reflective practices displayed in portfolio of dyads ST9 & ST10 further enhance their pedagogical development. Both students engage in critical self-assessment, identifying strengths and areas for improvement. This reflective process is vital, as research indicates that teacher beliefs and attitudes significantly impact the adoption of educational technologies [49]. By emphasizing the alignment of computational goals with mathematical learning objectives, ST9 and ST10 highlight the necessity for clarity and coherence in lesson planning.

The dyad ST9 and ST10 focused on upper-secondary mathematics content using Python and EduBlocks to support functional reasoning and operations with complex numbers, and also were used to foster logical processes and systematic problem-solving, which supported research associating programming with enhanced mathematical understanding [11, 12, 50]. Their planning included explicit modelling of concepts and attention to hypothetical learning trajectories, demonstrating awareness of potential misconceptions and the sequencing of conceptual progression. Their use of Python as both a mathematical and didactic tool illustrates a shift from task performance to instructional orchestration, as conceptualised in Drijvers et al.'s instrumental orchestration theory [32].

They anticipated student misconceptions by outlining learning trajectories and designing progressive activities. Lesson reflections revealed how Python's potential for visualising abstract concepts stimulated the integration of computational thinking with curriculum standards. The dyad employed forms of orchestration such as Technical-demo and Guide-and-explain, managing classroom activity through pre-planned coding tasks and structured questioning sequences. These strategies align with existing models for supporting the integration of computational tools in instructional design [33].

5.2.2 Results of ST11&ST12 dyad

In contrast, the dyad ST11 and ST12 addressed lower-secondary content and employed Scratch, in conjunction with GeoGebra, to develop lessons on linear functions, the Pythagorean theorem, and isometries. Their lesson designs featured narrative-based tasks and visual simulations to foster engagement and reasoning. Although their portfolios contained less emphasis on formal algorithmic structures than those of ST9 and ST10, their activities showed strong alignment with principles of exploratory learning and embodied modelling. The Scratch tasks incorporated iteration, decomposition, and visualisation, advancing computational thinking through interactive and accessible representations of mathematical ideas. The use of the Discuss-the-screen and Work-and-walk-by orchestrations promoted interactive learning while maintaining mathematical focus. The integration of Scratch and GeoGebra into their lesson plans showcases a dynamic approach to teaching, facilitating student engagement through interactive problem-solving. This approach align with the findings of recent studies indicating that engaging with various technologies can enhance pre-service teachers' subject knowledge and attitudes towards technology [22].

The emphasis on collaborative learning strategies in ST11's lesson plans is indicative of a pedagogical philosophy that values active participation. The use of Scratch to visualize linear functions allows students to interact with mathematical concepts in a tangible way, thereby fostering deeper comprehension. This aligns with the educational theories advocating for the integration of technological tools to enhance student learning experiences.

Similarly, ST12's focus on the Pythagorean theorem and isometries reflects an innovative pedagogical approach. The incorporation of storytelling in conjunction with Scratch fosters an exploratory learning environment, encouraging critical thinking and problem-solving skills. The application of Socratic dialogue not only promotes student engagement but also aligns with current educational theories that advocate for dialogue-rich classrooms.

These students concentrated on linear functions, the Pythagorean Theorem, and isometries for lower secondary education. Scratch and GeoGebra were employed to create interactive activities. The lesson plans presented opportunities for students to explore geometric and algebraic structures in a visual manner, which aligned with research indicating that interactive and dynamic tools promote reasoning and engagement [2]. The reflections addressed possible misconceptions and emphasised the importance of precise instructions, clear objectives, and guided discovery.

5.2.3 Integration of Computational Thinking

Both portfolios underscore the significance of computational thinking in mathematics education, albeit through different lenses. The emphasis on algorithmic processes and systematic problem-solving in ST9 and ST10's work illustrates their understanding of the cognitive processes involved in programming and mathematics. Their approach aligns with the TPACK framework, as they effectively integrate technology to foster students' logical reasoning and problem-solving skills [51].

Conversely, ST11 and ST12 highlight interactive problem-solving and visualization, utilizing tools that encourage students to engage deeply with mathematical principles. Their work exemplifies the key components of computational thinking, including decomposition and pattern recognition, which are essential for mastering complex mathematical concepts. This is consistent with recent findings that suggest integrating technology in teacher education can significantly enhance pre-service teachers' professional knowledge and teaching effectiveness [21].

The results in second cycle demonstrated greater alignment with computational thinking principles, including algorithmic design, decomposition, and abstraction [10, 11, 12]. The ST9 & ST10 portfolio underscored systematic approaches through Python coding, while ST11 & ST12 capitalised on interactive tasks in Scratch to encourage iterative refinements. These strategies exemplified the ways in which digital tools can deepen learners' reasoning in mathematics and help foster competencies required in contemporary curricula [50, 13, 14].

5.2.4 Integration of Instrumental Orchestration

The lesson plans revealed evidence of strategies consistent with instrumental orchestration [30, 31, 32], which describes how teachers design and manage technology-rich lessons. Students adopted orchestrations such as "Technical-demo," "Discuss-the-screen," and "Guide-and-explain," and these approaches illustrated a structured use of digital environments to direct student thinking [33, 34]. These findings confirmed the importance of lesson planning that connects pedagogical goals with suitable technology, as advocated in previous studies [21, 22]. Reflective discussions, both written and oral, indicated that the integration of these orchestration methods encouraged the ongoing adaptation and refinement of instructional strategies.

Both dyads demonstrated evidence of reflective practice, including post-lesson evaluation, consideration of feedback, and re-examination of teaching strategies. The inclusion of hypothetical learning trajectories and anticipatory scaffolding confirms that simulation-based learning environments can contribute meaningfully to teacher development. This is consistent with Vermunt et al. [18], who argue that the quality of teacher learning is enhanced when professional development activities are sustained, contextually embedded, and include opportunities for dialogue and feedback.

The comparative analysis of both cycles indicates that structured simulation and collaborative design processes support deeper integration of technology and foster the development of professional teaching knowledge. While the first cycle enabled technological exploration, the second cycle facilitated pedagogical refinement and theoretical grounding. This evolution demonstrates the utility of design-based research as a methodological approach for the development and analysis of complex educational interventions involving technology [48]. The results thus offered evidence that well-designed interventions can promote pre-service teachers' adoption of ICT in ways that align with national guidelines and support the growth of computational thinking [44].

Further investigation is warranted to determine the extent to which these design principles translate to authentic classroom environments during the induction phase. Simulated settings provide important data on pedagogical intent and planning processes, but cannot replicate the full complexity of live teaching, including student diversity, time constraints, and institutional demands. Longitudinal studies are needed to trace the persistence and transformation of these practices as pre-service teachers transition to professional contexts.

6 Final Remarks

This two-cycle investigation examined how pre-service mathematics teachers engage with computational thinking and technological tools within the framework of a design-based research methodology. The comparative analysis of both implementation cycles evidenced a developmental trajectory in the articulation of technological, pedagogical, and content knowledge. The first cycle revealed exploratory engagement with digital resources, with limited alignment between technological tools and pedagogical design. While students demonstrated basic operational competence with platforms such as GeoGebra and Scratch, few produced instructional materials that systematically addressed computational thinking or reflected coherent instructional trajectories.

The subsequent revision of the course design in the second cycle introduced structured collaborative planning, simulated teaching episodes, and guided reflection. These modifications were informed by theoretical constructs associated with instrumental orchestration and TPACK, and enabled participants to more effectively coordinate digital tools with curricular objectives. The analysis of student portfolios and teaching simulations confirmed that design tasks requiring anticipatory reasoning, peer feedback, and iterative refinement support the development of instructional competence involving computational technologies.

Findings suggest that the cultivation of computational thinking in mathematics teacher education cannot be realised through technological exposure alone. Rather, such development necessitates structured pedagogical interventions that are embedded in authentic learning environments and that engage prospective teachers in designing, enacting, and critically evaluating teaching practices. The results also underscore the importance of integrating simulated classroom practice into coursework, not as an ancillary component, but as a central mechanism for fostering pedagogical reasoning and reflective practice.

Given the growing prominence of computational thinking in national and international curricular frameworks, including its designation as a transversal competence in Portuguese mathematics education, the role of teacher education in supporting this integration warrants continued attention. The present study demonstrates that coherent and theoretically grounded interventions within teacher education programmes can enhance pre-service teachers' capacity to mobilise digital tools in ways that are pedagogically productive and aligned with curricular aims.

Future research should focus on tracing the transfer and adaptation of these practices into authentic classroom environments, particularly during the induction period. While simulation enables approximation of teaching practice, it remains limited in its capacity to capture the contingent and contextual demands of school-based instruction. Longitudinal research is therefore required to document how early professional experiences mediate the sustained use and adaptation of digital tools and computational thinking in mathematics classrooms.

Finally, this study reaffirms the value of design-based methodologies in mathematics education research. Such approaches provide a productive framework for iteratively refining pedagogical practices, generating situated knowledge, and informing institutional development in teacher preparation.

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References

[1] Michèle Artigue. *The Future of Teaching and Learning Mathematics with Digital Technologies*, page 463–475. Springer US, 2009.

[2] Nur Afiqah Zakaria and Fariza Khalid. The benefits and constraints of the use of information and communication technology (ict) in teaching mathematics. *Creative Education*, 07(11):1537–1544, 2016. http://dx.doi.org/10.4236/CE.2016.711158.

[3] Kaushik Das. Role of ict for better mathematics teaching. *Shanlax International Journal of Education*, 7(4):19–28, September 2019. http://dx.doi.org/10.34293/education.v7i4.641.

[4] Amina Safdar. *Effectiveness Of Information And Communication Technology (Ict) In Mathematics At Secondary Level*. PhD thesis, International Islamic University Islamabad, Islamabad, Pakistan, 2013. http://prr.hec.gov.pk/jspui/handle/123456789/709.

[5] A.P. Canavarro, C. Mestre, D. Gomes, E. Santos, L. Santos, L. Brunheira, M. Vicente, M.J. Gouveia, P. Correia, P. Marques, and R.G. Espadeiro. Aprendizagens essenciais de matemática no ensino básico, 2021. https://www.dge.mec.pt/aprendizagens-essenciais-0.Accessed: 2025-04-29.

[6] J. Carvalho e Silva, C. Albuquerque, J. Almiro, C. Cruchinho, S. Carreira, P. Correia, A. Domingos, G. E. Espadeiro, N. Filipe, L. Gabriel, H. Martins, M.E.G. Martins, A. Rodrigues, and M. T. Santos. Aprendizagens essenciais de matemática a, 2023. https://www.dge.mec.pt/aprendizagens-essenciais-0.Accessed: 2025-04-29.

[7] Nibedita Boruah. Impact of ict in education. *International journal of health sciences*, page 1818–1822, April 2022. http://dx.doi.org/10.53730/ijhs.v6ns2.5397.

[8] Saïd Assar. *Information and Communications Technology in Education*, page 66–71. Elsevier, 2015. http://dx.doi.org/10.1016/B978-0-08-097086-8.92104-4.

[9] Xinran Wang. An overview of ict and educational change: Development, impacts, and factors. *Journal of Contemporary Educational Research*, 6(8):1–8, August 2022. http://dx.doi.org/10.26689/jcer.v6i8.4191.

[10] Ilham Muhammad, Husnul Khatimah Rusyid, Swasti Maharani, and Lilis Marina Angraini. Computational thinking research in mathematics learning in the last decade: A bibliometric review. *International Journal of Education in Mathematics, Science and Technology*, 12(1):178–202, October 2023. http://dx.doi.org/10.46328/ijemst.3086.

[11] Erick John Fidelis Costa, Livia Maria Rodrigues Sampaio Campos, and Dalton Dario Serey Guerrero. Computational thinking in mathematics education: A joint approach to encourage problem-solving ability. In *2017 IEEE Frontiers in Education Conference (FIE)*. IEEE, October 2017. http://dx.doi.org/10.1109/FIE.2017.8190655.

[12] Maria Kallia, Sylvia Patricia van Borkulo, Paul Drijvers, Erik Barendsen, and Jos Tolboom. Characterising computational thinking in mathematics education: a literature-informed delphi study. *Research in Mathematics Education*, 23(2):159–187, January 2021. http://dx.doi.org/10.1080/14794802.2020.1852104.

[13] Siri Krogh Nordby, Annette Hessen Bjerke, and Louise Mifsud. Primary mathematics teachers' understanding of computational thinking. *KI - Künstliche Intelligenz*, 36(1):35–46, February 2022. http://dx.doi.org/10.1007/s13218-021-00750-6.

[14] Janice Teresinha Reichert, Dante Augusto Couto Barone, and Milton Kist. Computational thinking in k-12: An analysis with mathematics teachers. *Eurasia Journal of Mathematics, Science and Technology Education*, 16(6), March 2020. http://dx.doi.org/10.29333/ejmste/7832.

[15] Everett M. Rogers, Arvind Singhal, and Quinlan. Margaret M. Diffusion of innovations. In Don W. Stacks, Michael B. Salwen, and Kristen C. Eichhorn, editors, *An Integrated Approach to Communication Theory and Research*, pages 415–433. Routledge, 3 2019.

[16] Shannon O. Driskell, Sarah B. Bush, Robert N. Ronau, Margaret L. Niess, Christopher R. Rakes, and David K. Pugalee. *Handbook of Research on Transforming Mathematics Teacher Education in the Digital Age*, chapter Mathematics education technology professional development: Changes over several decades., pages 107–136. IGI Global, 2016. http://mdsoar.org/handle/11603/14415.

[17] Jodie Hunter and Jenni Back. Facilitating sustainable professional development through lesson study. *Mathematics Teacher Education and Development*, 13(1):94—-114, March 2013. https://mted.merga.net.au/index.php/mted/article/view/48/150.

[18] Jan D. Vermunt, Maria Vrikki, Nicolette van Halem, Paul Warwick, and Neil Mercer. The impact of lesson study professional development on the quality of teacher learning. *Teaching and Teacher Education*, 81:61–73, May 2019. http://dx.doi.org/10.1016/j.tate.2019.02.009.

[19] D. Jake Follmer, Randall Groth, Jennifer Bergner, and Starlin Weaver. Theory-based evaluation of lesson study professional development: Challenges, opportunities, and lessons learned. *American Journal of Evaluation*, 45(2):292–312, August 2023. http://dx.doi.org/10.1177/10982140231184899.

[20] Feng Liu, Albert D. Ritzhaupt, Kara Dawson, and Ann E. Barron. Explaining technology integration in k-12 classrooms: a multilevel path analysis model. *Educational Technology Research and Development*, 65(4):795–813, October 2016. http://dx.doi.org/10.1007/s11423-016-9487-9.

[21] Merrilyn Goos, Anne Bennison, and Robin Proffitt-White. Sustaining and scaling up research-informed professional development for mathematics teachers. *Mathematics Teacher Education and Development*, 20(2):133–150, May 2018. https://mted.merga.net.au/index.php/mted/article/view/430/324.

[22] Helena Rocha. *Pre-service Teachers' Knowledge and the Use of Different Technologies to Teach Mathematics*, page 505–515. Springer Singapore, November 2021. http://dx.doi.org/10.1007/978-981-16-5063-5_42.

[23] Rongjin Huang and Rose Mary Zbiek. *Prospective Secondary Mathematics Teacher Preparation and Technology*, page 17–23. Springer International Publishing, October 2016. http://dx.doi.org/10.1007/978-3-319-38965-3_3.

[24] Matthew J. Koehler, Punya Mishra, and Kurnia Yahya. Tracing the development of teacher knowledge in a design seminar: Integrating content, pedagogy and technology. *Computers & Education*, 49(3):740–762, 2007. https://doi.org/10.1016/j.compedu.2005.11.012.

[25] Melike Yigit. A review of the literature: How pre-service mathematics teachers develop their technological, pedagogical, and content knowledge. *International Journal of Education in Mathematics, Science and Technology*, 2(1), March 2014. http://dx.doi.org/10.18404/IJEMST.96390.

[26] S. Aslı Özgün-Koca, Michael S. Meagher, and Michael Todd Edwards. Preservice teachers' emerging tpack in a technology-rich methods class. *The Mathematics Educator*, 19(2):10–20, 2010. https://openjournals.libs.uga.edu/tme/article/view/1939/1844.

[27] John Ranellucci, Joshua M. Rosenberg, and Eric G. Poitras. Exploring pre-service teachers' use of technology: The technology acceptance model and <scp>expectancy–value</scp> theory. *Journal of Computer Assisted Learning*, 36(6):810–824, June 2020. http://dx.doi.org/10.1111/jcal.12459.

[28] Adedayo Olayinka Theodorio. Examining the support required by educators for successful technology integration in teacher professional development program. *Cogent Education*, 11(1), January 2024. http://dx.doi.org/10.1080/2331186X.2023.2298607.

[29] Jean-Stéphane Dhersin, HG Kaper, Wilfred Ndifon, Christiane Rousseau, and Günter M Ziegler. *Mathematics for action: supporting science-based decision-making*. United Nations Educational, Scientific and Cultural Organization, 2022. https://unesdoc.unesco.org/ark:/48223/pf0000380883.locale=en.pdf. Accessed: 2025-04-29.

[30] Corinne Thatcher Day. Expectancy value theory as a tool to explore teacher beliefs and motivations in elementary mathematics instruction. *lnternational Electronic Journal of Elementary Education*, 13(2):169–182, January 2021. http://dx.doi.org/10.26822/IEJEE.2021.182.

[31] Paul Drijvers, Michiel Doorman, Peter Boon, and Sjef van Gisbergen. Instrumental orchestration: theory and practice. In *Proceedings of the sixth congress of the European Society for Research in Mathematics Education*, pages 1349–1358, 2010. https://hal.science/hal-02182374.

[32] Paul Drijvers, Sebastian Grauwin, and Luc Trouche. When bibliometrics met mathematics education research: the case of instrumental orchestration. *ZDM*, 52(7):1455–1469, May 2020. http://dx.doi.org/10.1007/s11858-020-01169-3.

[33] Qi Tan and Zhiqiang Yuan. A professional development course inviting changes in pre-service mathematics teachers' integration of technology into teaching: the lens of instrumental orchestration. *Humanities and Social Sciences Communications*, 11(1), July 2024. http://dx.doi.org/10.1007/s11858-013-0535-1.

[34] Paul Drijvers, Sietske Tacoma, Amy Besamusca, Michiel Doorman, and Peter Boon. Digital resources inviting changes in mid-adopting teachers' practices and orchestrations. *ZDM*, 45(7):987–1001, August 2013. http://dx.doi.org/10.1007/s11858-013-0535-1.

[35] Matheus Souza de Almeida and Elisângela Bastos de Mélo Espíndola. Online instrumental orchestration: perspectives for the initial education of pre-service mathematics teachers in the context of the supervised teaching practice. *Em Teia | Revista de Educação Matemática e Tecnológica Iberoamericana*, 14(1):280, April 2023. http://dx.doi.org/10.51359/2177-9309.2023.257099.

[36] Rosilângela Lucena, Verônica Gitirana, and Luc Trouche. Teacher education for integrating resources in mathematics teaching: contributions from instrumental meta-orchestration. *The Mathematics Enthusiast*, 19(1):187–221, January 2022. http://dx.doi.org/10.54870/1551-3440.1549.

[37] Ruya Savuran and Hatice Akkoç. Examining pre-service mathematics teachers' use of technology from a sociomathematical norm perspective. *International Journal of Mathematical Education in Science and Technology*, 54(1):74–98, October 2021. http://dx.doi.org/10.1080/0020739X.2021.1966529.

[38] Francisco Eteval da Silva Feitosa, Sonia Barbosa Camargo Igliori, and Luc Trouche. The teaching professional knowledge formation by an instrumental meta-orchestration. *PNA. Revista de Investigación en Didáctica de la Matemática*, 18(1):35–56, October 2023. http://dx.doi.org/10.30827/pna.v18i1.25961.

[39] Gülay Bozkurt and Candas Uygan. Lesson hiccups during the development of teaching schemes: a novice technology-using mathematics teacher's professional instrumental genesis of dynamic geometry. *ZDM*, 52(7):1349–1363, July 2020. http://dx.doi.org/10.1007/s11858-020-01184-4.

[40] Dustin Schiering, Stefan Sorge, Steffen Tröbst, and Knut Neumann. Course quality in higher education teacher training: What matters for pre-service physics teachers' content knowledge development? *Studies in Educational Evaluation*, 78:101275, September 2023. http://dx.doi.org/10.1016/j.stueduc.2023.101275.

[41] Ali Bicer and Robert M. Capraro. Longitudinal effects of technology integration and teacher professional development on students' mathematics achievement. *EURASIA Journal of Mathematics, Science and Technology Education*, 13(3), December 2016. http://dx.doi.org/10.12973/EURASIA.2017.00645A.

[42] Merrilyn Goos. *Technology Integration in Secondary School Mathematics: The Development of Teachers' Professional Identities*, page 139–161. Springer Netherlands, September 2013. http://dx.doi.org/10.1007/978-94-007-4638-1_7.

[43] Donna F. Berlin and Arthur L. White. A longitudinal look at attitudes and perceptions related to the integration of mathematics, science, and technology education. *School Science and Mathematics*, 112(1):20–30, January 2012. http://dx.doi.org/10.1111/J.1949-8594.2011.00111.X.

[44] Vincent Geiger. *LEARNING MATHEMATICS WITH TECHNOLOGY FROM A SOCIAL PERSPECTIVE: A STUDY OF SECONDARY STUDENTS' INDIVIDUAL AND COLLABORATIVE PRACTICES IN A TECHNOLOGICALLY RICH MATHEMATICS CLASSROOM*. PhD thesis, University of Queensland Library, 2008. http://dx.doi.org/10.14264/178520.

[45] José Manuel Dos Santos, Jaime Carvalho e Silva, and Zsolt Lavicza. Geogebra classroom na formação inicial de professores de matemática. In Ana Amélia A. Carvalho, Eliane Schlemmer, Manuel Area, Célio Gonçalo Marques, Idalina Lourido Santos, Daniela Guimarães, Sónia Cruz, Idalina Moura, Carlos Sousa Reis, and Piedade Vaz Rebelo, editors, *Atas do 6º Encontro Internacional sobre Jogos e Mobile Learning*, pages 162–172, Coimbra, May 2024. Universidade de Coimbra, Centro de Estudos Interdisciplinares. https://hdl.handle.net/10316/115228.

[46] Ana Isabel Rosendo and Jaime Carvalho e Silva. Computers in mathematics education - an experience. In Fife and L. Husch, editors, *Electronic Proceedings of the 7th Annual International Conference on Technology in Collegiate Mathematics*, November 1994.

[47] Susan McKenney and Thomas C. Reeves. *Conducting Educational Design Research*. Routledge, 11 2018.

[48] Arthur Bakker. *Design Research in Education: A Practical Guide for Early Career Researchers*. Routledge, 7 2018.

[49] Shannon O Driskel, Sarah B Bush, Margaret L Niess, David Pugalee, Christopher R Rakes, and Robert N Ronau. Research in mathematics educational technology: Trends in professional development over 40 years of research. In T. G. Bartell, K. N. Bieda, R. T. Putnam, K. Bradfield, and H. Dominguez, editors, *North American Chapter of the International Group for the Psychology of Mathematics Education*, pages 656–662, East Lansing, MI: Michigan State University, June 2015. PME-NA, ACM Press. https://files.eric.ed.gov/fulltext/ED584325.pdf.

[50] Kristin Parve and Mart Laanpere. *Symbiotic Approach of Mathematical and Computational Thinking*, page 184–195. Springer Nature Switzerland, 2023. http://dx.doi.org/10.1007/978-3-031-43393-1_18.

[51] Shuchi Grover and Roy Pea. Computational thinking: A competency whose time has come. *Computer science education: Perspectives on teaching and learning in school*, 19:1257–1258, 2018.

Appendix - Plan used in Second Research Cycle

Table 2: Plan of MCEM course in 2023/2024

Session (n.)	Themes	Time (h)
1	Computational means to support proof in Geometry: GeoGebra and GeoGebra Discovery.	2
2	The impact and uses of the computer in the teaching and curriculum of Mathematics Part I	3
3	Computational means of proof support in Analysis and Algebra: Math Solver (`https://www.geogebra.org/solver?i=2%20x%2B1%3D5`), Wolfram Alpha.	2
4	Presentation of the Assymptote platform and exploration of the different types of automatic feedback available.	2
5	The Impact and Uses of the Computer in the Teaching and Curriculum of Mathematics Part II	3
6	Exploring the potential of various platforms for student assessment. Virtual classes: GeoGebra Classroom and other similar platforms.	2
7	Exploration of tools for learning Mathematics Part I.	3
8	Brief Introduction to Scratch Programming.	2
9	Computational Thinking. Computing and Education. The programming language is logo.	3
10	GeoGebra commands and tools for Statistics and Probabilities. The spreadsheet, in particular in GeoGebra.	2
11	The vision of the renewal of the curriculum in Basic and Secondary Education regarding the use of technological resources.	3
12	Introduction to programming in Python. Python by blocks. Exploration of Visual Python and Python in GeoGebra.	2
13	The vision of the renewal of the curriculum in Basic and Secondary Education regarding the use of technological resources. The role of Computational Thinking.	3
14	Python programming: in online compilers and in the graphing calculator.	2
15	Exploration of tools for learning Mathematics Part II.	3
16	3D Modeling and Printing	2
17	Presentation of 1st simulated classes - Theme 3	3
18	Presentation of 2nd simulated classes - Theme 3	2
19	Tasks involving technology for teaching and learning Mathematics, connections between various intra and extra Mathematics domains. Augmented reality as a feature.	2
20	Presentation of 1st simulated classes - Theme 2	2
21	Planning a Task that uses educational robotics for teaching and learning a mathematical topic.	2
22	Presentation of 2nd simulated classes - Theme 2	3
23	Analysis and self-hetero evaluation of the work carried out by the students.	2
24	Subjects related to the pedagogical internship and the year of induction.	3